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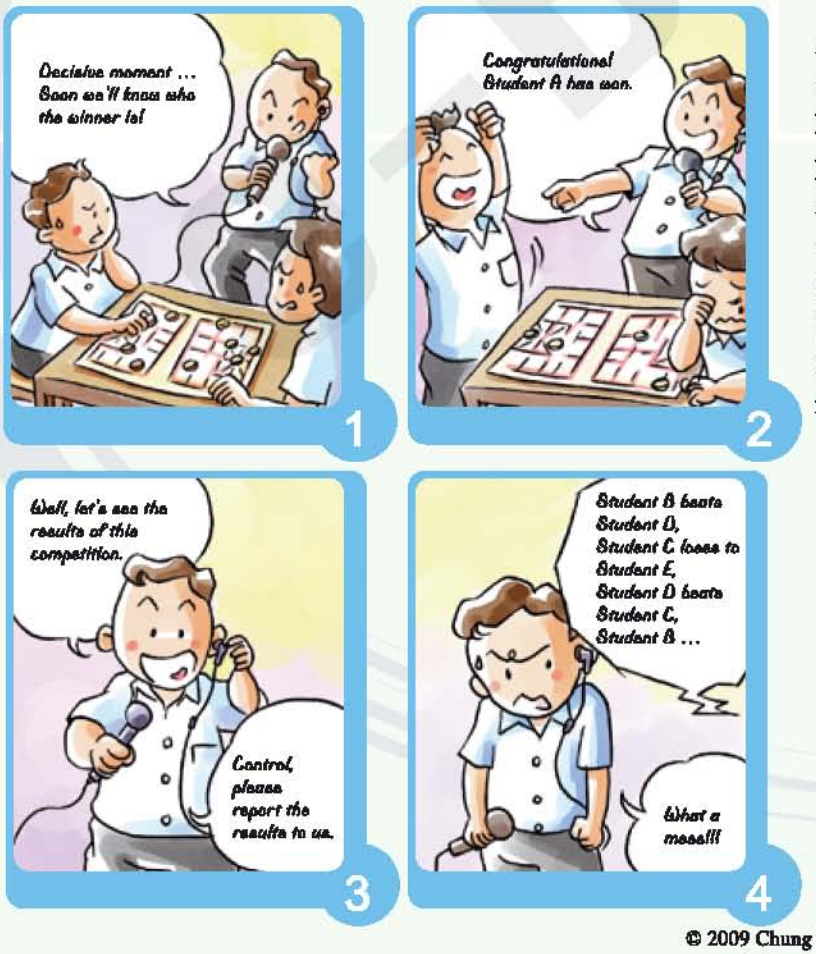
Chapter 9

Algebraic Fractions and Formulae

Learning Objectives

After completing this chapter, you will be able to

- master the operations of algebraic fractions.
- know some common formulae and substitute values into formulae.
- perform change of subject of a formula.



A school holds a chess competition. Every participant has to play against all other participants for a round each. It is given that a total of 10 rounds are required to be held if there are 5 participants. If there are altogether n participants and T rounds, write down the relation between T and n.

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Preview

[Basic techniques required for this chapter.]

Basic Technique

Methods of factorization

1. Taking out common factor
   e.g. $ab + ac = a(b + c)$

2. Grouping terms
   e.g. $ax + ay + bx + by = a(x + y) + b(x + y)$
   $= (x + y)(a + b)$

3. Using identities
   e.g. **(a)** $x^2 - 9 = x^2 - 3^2$
   $= (x + 3)(x - 3)$ ◂ $a^2 - b^2 \equiv (a + b)(a - b)$

   (b) $x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$
   $= (x + 5)^2$ ◂ $a^2 + 2ab + b^2 \equiv (a + b)^2$

   (c) $x^2 - 8x + 16 = x^2 - 2(x)(4) + 4^2$
   $= (x - 4)^2$ ◂ $a^2 - 2ab + b^2 \equiv (a - b)^2$

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9.1 Simple Algebraic Fractions

A Introduction to algebraic fractions

If P and Q are polynomials and $Q \neq 0$, then $P \div Q$ can be written as $\dfrac{P}{Q}$.

e.g. $4c \div 3ab = \dfrac{4c}{3ab}$, $(12x + 3) \div 4 = \dfrac{12x + 3}{4}$

In the expression $\dfrac{P}{Q}$, if the denominator Q involves variables, this expression is known as an **algebraic fraction**.

e.g. $\dfrac{4c}{3ab}$, $\dfrac{5}{x + 2}$ and $\dfrac{3}{y}$ are algebraic fractions, but $\dfrac{12x + 3}{4}$ is not.

Note: From now on in this chapter, we assume the values of denominators in algebraic fractions are not 0. For example, in the algebraic fraction $\dfrac{1}{x + 1}$, $x + 1$ is not equal to 0, i.e. $x \neq -1$.

algebraic fraction 代數分式

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B Simplification of algebraic fractions

The concepts of algebraic fractions are similar to those of fractions.

We can simplify a fraction by cancelling out the common factor of the numerator and denominator.

e.g. $\dfrac{32}{48}=\dfrac{2\times\cancel{16}}{3\times\cancel{16}}=\dfrac{2}{3}$

In an algebraic fraction, if the numerator and denominator have a common factor, we can simplify the algebraic fraction by cancelling out the common factor.

e.g. $\dfrac{2a}{5a}=\dfrac{2\cancel{a}}{5\cancel{a}}=\dfrac{2}{5}$

Additional Example 9.1

Example 9.1 Simplifying algebraic fractions with monomials as denominators

Simplify the following expressions.

(a) $\dfrac{4p^2q}{12pq^3}$ **(b)** $\dfrac{4ax+4ay}{ax}$

Solution

(a) $\dfrac{4p^2q}{12pq^3}=\dfrac{\cancel{4pq}(p)}{\cancel{4pq}(3q^2)}$

$=\underline{\underline{\dfrac{p}{3q^2}}}$ ◂ Cancel out the common factor $4pq$.

(b) $\dfrac{4ax+4ay}{ax}=\dfrac{4\cancel{a}(x+y)}{\cancel{a}x}$ ◂ Factorize the numerator.

$=\underline{\underline{\dfrac{4(x+y)}{x}}}$ ◂ Cancel out the common factor a.

Classwork 9.1 Solution

Simplify the following expressions.

(a) $\dfrac{6a^2y^3}{2a^3y}$

(b) $\dfrac{12xy-8x^2}{4x}$

Additional Example 9.2

Example 9.2 Simplifying algebraic fractions with binomials as denominators

Simplify the following expressions.

(a) $\dfrac{3y^2-6y}{8-4y}$ **(b)** $\dfrac{9x^2-1}{3x-1}$

Classwork 9.2 Solution

Simplify the following expressions.

(a) $\dfrac{x^3-2x^2}{4-2x}$

(b) $\dfrac{12x^2-3}{2x+1}$

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Solution

(a) $\dfrac{3y^2-6y}{8-4y}=\dfrac{3y\cancel{(y-2)}}{-4\cancel{(y-2)}}$ ◂ Factorize both denominator and numerator, of which $8-4y=-(4y-8)=-4(y-2)$.

$=\underline{\underline{-\dfrac{3y}{4}}}$ ◂ Cancel out the common factor $(y-2)$.

(b) $\dfrac{9x^2-1}{3x-1}=\dfrac{(3x)^2-1^2}{3x-1}$

$=\dfrac{(3x+1)\cancel{(3x-1)}}{\cancel{3x-1}}$ ◂ Factorize the numerator by the formula $a^2-b^2\equiv(a+b)(a-b)$.

$=\underline{\underline{3x+1}}$ ◂ Denominator equals 1 after cancelling out the common factor $(3x-1)$.

C Multiplication and division of algebraic fractions

The multiplication and division of algebraic fractions are similar to those of fractions. Before performing the operations of multiplication or division, we should factorize the numerator and denominator first, and then cancel out the common factors.

Additional Example 9.3

Example 9.3 Multiplication of algebraic fractions

Simplify the following expressions.

(a) $\dfrac{6x}{5y}\times\dfrac{y^2}{3x}$

(b) $\dfrac{-2x}{3x+3y}\times\dfrac{4x+4y}{-5x}$

Solution

(a) $\dfrac{6x}{5y}\times\dfrac{y^2}{3x}=\dfrac{2(3x)}{5y}\times\dfrac{y^2}{3x}$

$=\underline{\underline{\dfrac{2y}{5}}}$ ◂ Cancel out the common factors $3x$ and y.

(b) $\dfrac{-2x}{3x+3y}\times\dfrac{4x+4y}{-5x}=\dfrac{2(-x)}{3(x+y)}\times\dfrac{4(x+y)}{5(-x)}$ ◂ Factorize $(4x+4y)$ and $(3x+3y)$.

$=\underline{\underline{\dfrac{8}{15}}}$ ◂ Cancel out the common factors $-x$ and $(x+y)$.

Classwork 9.3 Solution

Simplify the following expressions.

(a) $\dfrac{-4a}{3b}\times\dfrac{12b^3}{-16a^2}$

(b) $\dfrac{6a}{5a-5b}\times\dfrac{2a-2b}{3a}$

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Additional Example 9.4

Example 9.4 Division of algebraic fractions

Simplify the following expressions.

(a) $\dfrac{5a^2}{2b^2} \div \dfrac{10a^3}{-4b}$

(b) $\dfrac{2x-4y}{3z^2} \div \dfrac{2y-x}{2z} \times 6z$

Solution

(a) $\dfrac{5a^2}{2b^2} \div \dfrac{10a^3}{-4b}$

$= \dfrac{5a^2}{2b^2} \times \dfrac{-4b}{10a^3}$

$= \dfrac{5a^2}{b(2b)} \times \dfrac{-2(2b)}{2a(5a^2)}$

$= \underline{\underline{-\dfrac{1}{ab}}}$ ◂ Cancel out the common factors $5a^2$, 2 and $2b$.

(b) $\dfrac{2x-4y}{3z^2} \div \dfrac{2y-x}{2z} \times 6z$

$= \dfrac{2x-4y}{3z^2} \times \dfrac{2z}{2y-x} \times 6z$

$= \dfrac{2(x-2y)}{3z^2} \times \dfrac{2z}{-(x-2y)} \times 6z$ ◂ Factorize $(2x-4y)$; $2y-x=-(x-2y)$

$= \underline{\underline{-8}}$ ◂ Cancel out the common factors $(x-2y)$, 3 and z^2.

Classwork 9.4

Solution

Simplify the following expressions.

(a) $\dfrac{3x^2y}{5x^3y} \div \dfrac{-15x^2}{3y}$

(b) $\dfrac{8a}{6c-2d} \div \dfrac{2a^2}{5d-15c} \times 2a$

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Skills Upgrading Corner 9.1

Solution

Simplify the following expressions.

(a) $\dfrac{x-y}{3x+6y} \times \dfrac{2y+x}{(x-y)^2}$

(b) $\dfrac{h+k}{h^2-k^2} \div \dfrac{4k}{2h-2k}$

(c) $\dfrac{2a}{3b} \times \dfrac{9b^2}{5a^3} \div \dfrac{b}{15a}$

(d) $\dfrac{(5m-6)^2}{18-15m} \div \dfrac{10mn-12n}{4m^2} \times \dfrac{24mn}{16n^2}$

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Exercise 9A

Level 1

Simplify the following expressions. (**1 – 4**)

1. **(a)** $\dfrac{6t}{4tu}$ **(b)** $\dfrac{25x}{5x^2}$ **(c)** $\dfrac{15a^2b}{9b}$ **(d)** $\dfrac{8x^2y}{36y^2z}$

2. **(a)** $\dfrac{ab+bc}{3b}$ **(b)** $\dfrac{mp}{mn-mp}$ **(c)** $\dfrac{5ab+10a}{3a}$ **(d)** $\dfrac{a^2}{a^2b-ab^2}$

3. **(a)** $\dfrac{ab-ac}{ab+ac}$ **(b)** $\dfrac{3ay+a}{ay-3a}$ **(c)** $\dfrac{2ax-2bx}{5b-5a}$ **(d)** $\dfrac{2x-6y}{3y-x}$

4. **(a)** $\dfrac{k-m}{k^2-km}$ **(b)** $\dfrac{5a+15}{6a^2+18a}$ **(c)** $\dfrac{4-x^2}{2-x}$ **(d)** $\dfrac{-x^2+1}{x+1}$

Level 2

Simplify the following expressions. (**5 – 10**)

5. **(a)** $\dfrac{a^3}{2b}\times\dfrac{b^2}{a}$ **(b)** $\dfrac{15a^2b}{7ac^2}\times\dfrac{14b^3c}{5a^2b^2}$ **(c)** $\dfrac{8x^2y^2}{25xz^3}\times\dfrac{20yz}{16y^2z^2}$

6. **(a)** $\dfrac{6x^2}{y}\div\dfrac{2x}{y^2}$ **(b)** $\dfrac{9x^2y}{3y^2z}\div\dfrac{6xz^2}{2yz}$ **(c)** $\dfrac{10a^4b}{9a^2c^2}\div\dfrac{15b^2c}{8bc^3}$

7. **(a)** $\dfrac{3x-3y}{4x}\times\dfrac{3ax}{ax-ay}$ **(b)** $\dfrac{4ab}{5a+5b}\times\dfrac{3b+3a}{2a^2}$ **(c)** $\dfrac{3m-2n}{(5n+m)^2}\times\dfrac{2m+10n}{4n-6m}$

8. **(a)** $\dfrac{2p-10r}{r^2}\div\dfrac{3p-15r}{6pr}$ **(b)** $\dfrac{ax-a}{bx-3b}\div\dfrac{c-cx}{3d-dx}$ **(c)** $\dfrac{x^2-y^2}{2x-2y}\div\dfrac{3x+3y}{6x}$

9. **(a)** $\dfrac{6x^2}{7y^3}\div\dfrac{3x}{14z}\times\dfrac{9y^2}{2z}$ **(b)** $\dfrac{12k}{\ell^2}\times\dfrac{2\ell}{3k^2}\div\dfrac{2k}{\ell}$ **(c)** $\dfrac{15c^2}{d^3}\div\dfrac{5c}{d}\times\dfrac{d^2}{9c}$

10. **(a)** $\dfrac{8x}{5y}\times\dfrac{10x-2}{16x}\div\dfrac{4-20x}{10y}$ **(b)** $\dfrac{ab+3b}{a}\times\dfrac{5+5b}{a+3}\div(ab+a)$ **(c)** $\dfrac{(4m-1)^2}{15-5n}\times\dfrac{n^2-3^2}{n-4mn}\div(4m-1)$

11. **(a)** Factorize $2ac+bd+ad+2bc$.
(b) From the result of **(a)**, simplify $\dfrac{2ac+bd+ad+2bc}{a^2-b^2}$.

12. **(a)** Factorize $2y^2+20y+50$.
(b) From the result of **(a)**, simplify $\dfrac{2y^2+20y+50}{w^2}\times\dfrac{6w}{2y+10}$.

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9.2 Addition and Subtraction of Algebraic Fractions

The addition and subtraction of algebraic fractions are similar to those of fractions. When the denominators in algebraic fractions are different, we need to convert them into a common denominator before addition or subtraction of their numerators.

e.g. Before simplifying $\frac{1}{2x}+\frac{1}{3x}$, we can convert the denominators into a common denominator by multiplying the denominator and numerator of each algebraic fraction with a non-zero number.

$$\frac{1}{2x}=\frac{1}{2x}\cdot\frac{3}{3}=\frac{3}{6x}$$

$$\frac{1}{3x}=\frac{1}{3x}\cdot\frac{2}{2}=\frac{2}{6x}$$

Then we can add the numerators together, i.e.

$$\begin{aligned}\frac{1}{2x}+\frac{1}{3x}&=\frac{3}{6x}+\frac{2}{6x}\\&=\frac{3+2}{6x}\\&=\frac{5}{6x}\end{aligned}$$

Additional Example 9.5

Example 9.5 Addition and subtraction of algebraic fractions with a common denominator

Simplify the following expressions.

(a) $\frac{1}{3x}+\frac{5}{3x}$ **(b)** $\frac{1}{x-1}-\frac{x}{x-1}$

Solution

(a) $$\begin{aligned}\frac{1}{3x}+\frac{5}{3x}&=\frac{6}{3x}\\&=\underline{\underline{\frac{2}{x}}}\end{aligned}$$

If the denominators of two algebraic fractions are the same, the addition and subtraction of numerators can be performed directly.

(b) $$\begin{aligned}\frac{1}{x-1}-\frac{x}{x-1}&=\frac{1-x}{x-1}\\&=\frac{-(x-1)}{x-1}\\&=\underline{\underline{-1}}\end{aligned}$$

$1-x=-(x-1)$

Classwork 9.5 Solution

Simplify the following expressions.

(a) $\frac{2}{9y}+\frac{4}{9y}$

(b) $\frac{4}{y-2}-\frac{2y}{y-2}$

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Additional Example 9.6

Example 9.6 Addition and subtraction of algebraic fractions with denominators having different signs

Simplify the following expressions.

(a) $\dfrac{3}{a-3}+\dfrac{2}{3-a}$

(b) $\dfrac{3}{2x-1}-\dfrac{2x}{1-2x}$

Solution

(a)
$$\begin{aligned}\frac{3}{a-3}+\frac{2}{3-a}&=\frac{3}{a-3}+\frac{2}{-(a-3)}\\&=\frac{3}{a-3}-\frac{2}{a-3}\\&=\underline{\underline{\frac{1}{a-3}}}\end{aligned}$$

◀ $3-a=-(a-3)$

(b)
$$\begin{aligned}\frac{3}{2x-1}-\frac{2x}{1-2x}&=\frac{3}{2x-1}-\frac{2x}{-(2x-1)}\\&=\frac{3}{2x-1}+\frac{2x}{2x-1}\\&=\underline{\underline{\frac{3+2x}{2x-1}}}\end{aligned}$$

Classwork 9.6 Solution

Simplify the following expressions.

(a) $\dfrac{5}{3x-1}+\dfrac{2}{1-3x}$

(b) $\dfrac{3}{4-3y}-\dfrac{2y}{3y-4}$

Additional Example 9.7

Example 9.7 Addition and subtraction of algebraic fractions with different denominators

Simplify the following expressions.

(a) $\dfrac{1}{2x}-\dfrac{1}{5x}$

(b) $\dfrac{4}{3(x-1)}+\dfrac{1}{5(1-x)}$

Solution

(a)
$$\begin{aligned}\frac{1}{2x}-\frac{1}{5x}&=\frac{1}{2x}\cdot\frac{5}{5}-\frac{1}{5x}\cdot\frac{2}{2}\\&=\frac{5}{10x}-\frac{2}{10x}\\&=\underline{\underline{\frac{3}{10x}}}\end{aligned}$$

◀ If the denominators of two algebraic fractions are different, we have to convert them into a common denominator.

Classwork 9.7 Solution

Simplify the following expressions.

(a) $\dfrac{2}{3y}-\dfrac{2}{5y}$

(b) $\dfrac{3}{2(x-4)}+\dfrac{1}{3(4-x)}$

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(b) $\dfrac{4}{3(x-1)}+\dfrac{1}{5(1-x)}=\dfrac{4}{3(x-1)}\cdot\dfrac{5}{5}+\dfrac{1}{5(1-x)}\cdot\dfrac{3}{3}$

$=\dfrac{20}{15(x-1)}+\dfrac{3}{15(1-x)}$

$=\dfrac{20}{15(x-1)}+\dfrac{3}{15(-1)(x-1)}$

$=\dfrac{20}{15(x-1)}-\dfrac{3}{15(x-1)}$

$=\underline{\underline{\dfrac{17}{15(x-1)}}}$

Additional Example 9.8

Example 9.8 Addition and subtraction of integers and algebraic fractions

Simplify the following expressions.

(a) $1+\dfrac{3-y}{3y}$

(b) $\dfrac{5y+7}{y+3}-2$

Solution

(a) $1+\dfrac{3-y}{3y}=\dfrac{3y}{3y}+\dfrac{3-y}{3y}$ ◂ $1=\dfrac{3y}{3y}$

$=\dfrac{3y+3-y}{3y}$

$=\underline{\underline{\dfrac{2y+3}{3y}}}$

(b) $\dfrac{5y+7}{y+3}-2=\dfrac{5y+7}{y+3}-\dfrac{2(y+3)}{y+3}$ ◂ $2=\dfrac{2(y+3)}{y+3}$

$=\dfrac{5y+7-2(y+3)}{y+3}$

$=\dfrac{5y+7-2y-6}{y+3}$

$=\underline{\underline{\dfrac{3y+1}{y+3}}}$

Classwork 9.8

Solution

Simplify the following expressions.

(a) $\dfrac{1-2x}{4x}+2$

(b) $3+\dfrac{2y}{1-2y}$

Skills Upgrading Corner 9.2

Solution

Simplify the following expressions.

(a) $\dfrac{y}{2x}-\dfrac{3}{4x}$

(b) $\dfrac{m}{4(m-5)}+\dfrac{1}{5(m-5)}$

(c) $\dfrac{2r}{r-2s}-1$

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Exercise 9B

Level 1

Simplify the following expressions. (**1** – **4**)

1. **(a)** $\frac{1}{x}+\frac{3}{x}$ **(b)** $\frac{7}{6y}+\frac{5}{6y}$ **(c)** $-\frac{1}{8z}+\frac{7}{8z}$

2. **(a)** $\frac{3}{p+3}+\frac{2}{p+3}$ **(b)** $\frac{1}{1-q}-\frac{q}{1-q}$ **(c)** $-\frac{2}{r-2}+\frac{7r}{r-2}$

3. **(a)** $\frac{h}{h+k}+\frac{k}{h+k}$ **(b)** $\frac{2n-m}{n-3m}-\frac{3m}{n-3m}$ **(c)** $-\frac{k}{5(k+\ell)}-\frac{5-k}{5(k+\ell)}$

4. **(a)** $\frac{2}{x-1}+\frac{1}{1-x}$ **(b)** $\frac{4}{2y-1}-\frac{y}{1-2y}$ **(c)** $\frac{7z}{3z-2}+\frac{z+4}{2-3z}$

Level 2

Simplify the following expressions. (**5** – **10**)

5. **(a)** $\frac{2}{a}+\frac{1}{2a}$ **(b)** $\frac{3}{5b}-\frac{1}{b}$ **(c)** $\frac{1}{2c}-\frac{5}{6c}$

6. **(a)** $\frac{2}{3x}-\frac{1}{2x}$ **(b)** $\frac{1}{2y}+\frac{3}{7y}$ **(c)** $\frac{7}{5z}-\frac{4}{3z}$

7. **(a)** $\frac{2}{p-3}+\frac{3}{5(p-3)}$ **(b)** $\frac{q}{2(q-4)}-\frac{2}{q-4}$ **(c)** $-\frac{1}{2+r}-\frac{r}{2(r+2)}$

8. **(a)** $\frac{1}{2(h+1)}-\frac{3}{7(1+h)}$ **(b)** $\frac{k}{5(k-3)}-\frac{3}{2(3-k)}$ **(c)** $\frac{m}{3(2m+9)}+\frac{3}{2(2m+9)}$

9. **(a)** $\frac{a-1}{5a}+1$ **(b)** $\frac{2+3b}{4b}-2$ **(c)** $3-\frac{5-2c}{c+4}$

10. **(a)** $1-\frac{3r}{2(s+r)}$ **(b)** $-\frac{2s}{3t+s}+2$ **(c)** $-3u-\frac{7ut}{3-2t}$

9.3 Formulae and Substitution

A Formulae

We have learned the concept of formulae in Form 1.

> Formula is an algebraic equality to express the relation among variables.

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e.g. The following are two formulae in relation to a rectangle.

Perimeter of rectangle: $P = 2(\ell + b)$

Area of rectangle: $A = \ell b$

where ℓ is the length, b is the width, P is the perimeter and A is the area.

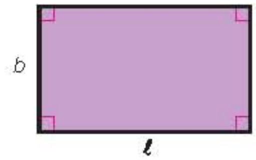


Figure 9.1

Note: In the same formula,

(i) the same variable can only be represented by the same letter;

(ii) the upper case and lower case of the same letter often represent different variables.

e.g. Area of trapezium $A = \frac{1}{2}(a+b)h$, where A represents the area of trapezium, while a represents the length of one of the parallel sides.

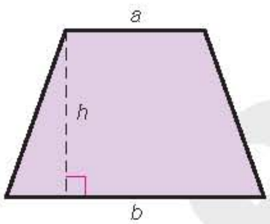
B Substitution

The process of replacing variables in a formula by some values is called substitution. If all variables, except a specific one, are substituted by some values, the value of the specific variable can be found.

Additional Example 9.9

Example 9.9 Finding unknowns in formulae by substitution

It is given that the formula for the area of a trapezium is $A = \frac{1}{2}(a+b)h$.



(a) If $a = 3$, $b = 5$ and $h = 4$, find the value of A.

(b) If $a = 9$, $h = 16$ and $A = 176$, find the value of b.

Solution

(a) $A = \frac{1}{2}(3+5)(4)$

$= \underline{\underline{16}}$

(b) $176 = \frac{1}{2}(9+b)(16)$

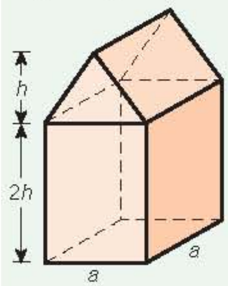
$9 + b = 22$

$b = \underline{\underline{13}}$

Classwork 9.9

Solution

In the figure, the volume of the solid is given by the formula $V = \frac{5}{2}a^2h$.



3D

(a) If $a = 4$ and $h = 6$, find the value of V.

(b) If $V = 630$ and $h = 7$, find the value of a.

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Additional Example 9.10

Example 9.10 Application of substitution

From a medical research, the pulse rate P bpm (number of heartbeats per minute) of a healthy person of a years old can stand is given by the following formula:

$$P = \frac{3}{4}(220 - a)$$

(a) If Gary is 20 years old, what is the pulse rate can he stand?

(b) After taking a *roller coaster* ride, Gary finds that the number of his heartbeats in 10 seconds is 28. Is he suitable to take the ride? Explain briefly.

Solution

(a) When $a = 20$,

$$P = \frac{3}{4}(220 - 20)$$
$$= 150$$

$\therefore$ The pulse rate that Gary can stand is 150 bpm.

(b) If the number of his heartbeats in 10 seconds is 28,

then his pulse rate $= 28 \times 6$ bpm
$= 168$ bpm
> 150 bpm

$\because$ This pulse rate exceeds the pulse rate Gary can stand.

$\therefore$ Gary is not suitable to take the ride.

Classwork 9.10

Solution

It is known that the cost $\$C$ of developing a computer game and distributing n sets of it is given by $C = 200\,000 + 30n$.

(a) What is the cost of distributing 4 000 sets of the game?

(b) If the price of each set of the game is \$90, in order to make a profit percentage of 80%,

(i) what should be the cost of each set of the game?

(ii) how many sets of the game should be distributed to reach this target?

Skills Upgrading Corner 9.3

Solution

1. It is given that $s = ut + \frac{1}{2}at^2$. If $s = 583$, $t = 11$ and $u = -2$, find the value of a.

Solution

2. Given that $ab - 3 = 4(a + b)$, find the value of b if $a = 3$.

Solution

3. Given that $k^2 = \frac{5 + 10x}{x - 4}$, find the value of x if $k = 5$.

Solution

4. It is given that the sum of all integers from 1 to n is $\frac{n(n+1)}{2}$, i.e. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$.

(a) Find $1 + 2 + 3 + \cdots + 50$.

(b) Find $1 + 2 + 3 + \cdots + 100$.

(c) Find $51 + 52 + 53 + \cdots + 100$.

(d) Find $21 + 22 + 23 + \cdots + 50$.

roller coaster 過山車

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Exercise 9C

Level 1

1. Given that $A=\frac{360°}{n}$, find the value of A if $n=8$.

2. Given that $y=\frac{1}{3}x(x+1)$, find the value of y if $x=30$.

3. Given that $P=2(\ell+b)$, find the value of P if $\ell=10$ and $b=5$.

4. Given that $A=\frac{1}{2}bh$, find the value of A if $b=9$ and $h=8$.

5. Given that $A=\pi r^2$, find the value of A if $\pi=\frac{22}{7}$ and $r=14$.

6. Given that $z=\frac{xy}{x+y}$, find the value of z if $x=10$ and $y=40$.

7. Given that $a+b+c=180$, find the value of b if $a=90$ and $c=56$.

8. It is given that $V=\ell bh$. If $V=120$, $b=4$ and $\ell=10$, find the value of h.

9. Given that $E=(n-2)\times 180$, find the value of n if $E=1\ 080$.

10. It is given that $P=\frac{A}{1+I}$. If $P=1\ 000$ and $I=0.08$, find the value of A.

Level 2

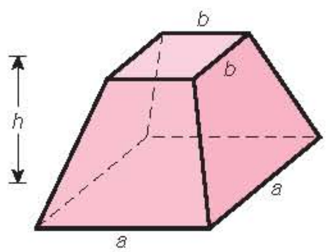
11. It is given that $A=P(1+rt)$. If $A=1\ 240$, $P=1\ 000$ and $r=0.06$, find the value of t.

12. It is given that $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. If $u=15$ and $v=30$, find the value of f.

13. Given that $a(x+2)=3x$, find the value of x if $a=6$.

14. Given that $A^2=\frac{1+3b}{2-b}$, find the value of b if $A=5$.

15. In the figure, the volume of the solid is given by
$V=\frac{1}{3}h(a^2+ab+b^2)$.
(a) If $h=8$, $a=12$ and $b=6$, find the value of V.
(b) If $V=818$, $a=15$ and $b=8$, find the value of h.



3D

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16. There were level one and level two questions in a quiz. If a participant answered x level one questions and y level two questions correctly within the time allowed, a score M would be obtained, where $M = 10x + 20y$.

(a) If Joyce answered 5 level one questions and 2 level two questions correctly, what would be her score?

(b) If Mary answered 3 level one questions correctly and obtained a score of 110, how many level two questions did she answer correctly?

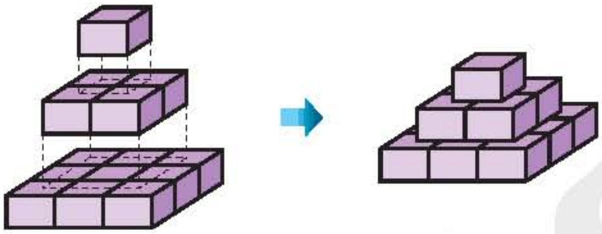
17. It is given that each interior angle of a regular n-gon is given by $I = \dfrac{(n-2)\times 180°}{n}$.

(a) Find the size of each interior angle of a regular 10-gon.

(b) Find the size of each interior angle of a regular 12-gon.

(c) If the size of each interior angle of a regular polygon is 162°, how many sides does this polygon have?

18. A tower is composed of a stack of ***bricks*** with 1 brick in the top layer. There are 4 (i.e. 2^2) bricks in the 2nd layer, 9 (i.e. 3^2) bricks in the 3rd layer, and so on.



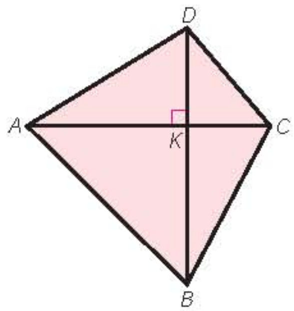
It is known that the sum of the squares of all integers from 1 to n is $\dfrac{n(n+1)(2n+1)}{6}$, i.e.
$1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

(a) If there are altogether 7 layers in the tower, how many bricks are there?

(b) If the tower in **(a)** is extended to 10 layers, how many additional bricks are required?

(c) Are 700 bricks enough for building a tower with 13 layers? Explain briefly.

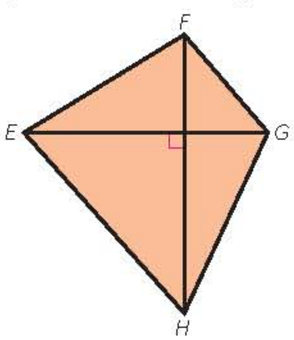
19. In the figure, $ABCD$ is a quadrilateral, where AC is perpendicular to BD at K. It is given that $AC = a$, $BD = b$, $AK = x$ and $BK = y$.

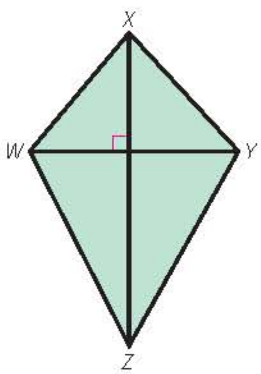


(a) Find the areas of ΔABK, ΔBCK, ΔCDK and ΔDAK in terms of a, b, x and y.

(b) Find the formula for the area S of $ABCD$.

(c) The following figures are quadrilaterals $EFGH$ and $WXYZ$, where $EG = 7$, $FH = 8$, $WY = 6$ and $XZ = 9$. Determine which quadrilateral is larger.





brick 磚

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9.4 Change of Subject of a Formula

In a formula, when a variable is expressed in terms of other variables, it is called the **subject of the formula**.

e.g. Consider the formula for the perimeter of rectangle.

$$P = 2(\ell + b)$$

Subject

Since variable P is represented by the expression in ℓ and b, P is the subject of this formula.

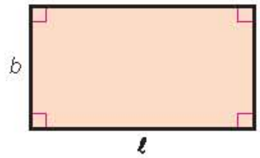

Figure 9.2

Class Activity 9.1

Aim: To learn the concept of changing subject of a formula

Amy is planning a trip to Malaysia, and she has worked out her expected spending for the trip as follows.

Return ticket between Hong Kong and Malaysia	\$1800 / ticket
Airport departure tax	\$700 / head
Four-star hotel (including breakfast)	\$550 / night
Meals and shopping	\$300 / day

(a) Suppose the total spending of a trip for n days is \$$P$. Write down a formula with P as the subject.
[Hint: A 5-day trip needs 4-night accommodation.]

Show Ans.

(b) Substitute each value of P in the table into the formula in **(a)**. Find the number of days n of a trip which can be planned with different spending.

Show Ans.

P	5 350	6 200	7 050	7 900	8 750
n					

(c) Make n the subject of the formula in **(a)** (i.e. rewrite the formula as 'n is equal to an expression in P').

Show Ans.

subject of the formula 公式的主項

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(d) If Amy has \$11 000, by using the formula in **(c)**, she can plan her trip for a maximum of ____________ days. Show Ans.

(e) **(b) / (d)** is the quicker way to find the number of days n. Show Ans.

Now I see ...
We can make different variables as the subject of the same formula.

In fact, the following formulae can be interchanged with each other, with different subjects.

$$P = 1\,950 + 850n \quad \xleftrightarrow{\text{Change of Subject}} \quad n = \frac{P - 1\,950}{850}$$

P is the subject — n is the subject

Additional Example 9.11

Example 9.11 Change of subjects of simple formulae

Make x the subject of each of the following formulae.

(a) $y = 3x + 1$

(b) $y = \dfrac{2x - 5}{3}$

Solution

(a)
$$\begin{aligned} y &= 3x + 1 \\ y - 1 &= 3x \\ 3x &= y - 1 \\ \therefore \quad x &= \underline{\underline{\frac{y-1}{3}}} \end{aligned}$$

◂ Move the term with x to the left-hand side of the equality sign.

(b)
$$\begin{aligned} y &= \frac{2x - 5}{3} \\ 3y &= 2x - 5 \\ 3y + 5 &= 2x \\ 2x &= 3y + 5 \\ \therefore \quad x &= \underline{\underline{\frac{3y+5}{2}}} \quad \left(\text{or } \underline{\underline{x = \frac{3y}{2} + \frac{5}{2}}}\right) \end{aligned}$$

◂ Move the term with x to the left-hand side of the equality sign.

◂ $\dfrac{3y+5}{2} = \dfrac{3y}{2} + \dfrac{5}{2}$

Classwork 9.11

Solution

Make x the subject of each of the following formulae.

(a) $y = \dfrac{5 + 4x}{3}$

(b) $y - 5 = \dfrac{2x + 3}{6}$

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Additional Example 9.12

Example 9.12 Change of subject of a formula involving brackets

Make r the subject of the formula $A = P(1+rt)$.

Solution

$$A = P(1+rt)$$
$$A = P + Prt \quad \text{◂ Remove brackets}$$
$$A - P = Prt$$
$$Prt = A - P \quad \text{◂ Move the term with } r \text{ to the left-hand side of the equality sign.}$$
$$\therefore \quad \underline{\underline{r = \frac{A-P}{Pt}}} \quad \left(\text{or } \underline{\underline{r = \frac{A}{Pt} - \frac{1}{t}}}\right)$$

Classwork 9.12 Solution

Make b the subject of each of the following formulae.

(a) $P = 2(\ell + b)$

(b) $A = \frac{1}{2}(a+b)h$

Additional Example 9.13

Example 9.13 Change of subject of a more complicated formula

Make x the subject of the formula $y = \frac{1+x}{2-x}$.

Solution

$$y = \frac{1+x}{2-x}$$
$$y(2-x) = 1+x$$
$$2y - xy = 1+x$$
$$2y - 1 = xy + x$$
$$2y - 1 = x(y+1) \quad \text{◂ Factorization}$$
$$\therefore \quad \underline{\underline{x = \frac{2y-1}{y+1}}}$$

Classwork 9.13 Solution

Make x the subject of each of the following formulae.

(a) $y = \frac{3x}{x+3}$

(b) $y = \frac{4+x}{3-x}$

Additional Example 9.14

Example 9.14 Change of subject involving the conversion into a common denominator

Make a the subject of the formula $\frac{2}{a} + \frac{1}{b} = \frac{1}{c}$.

Solution

$$\frac{2}{a} + \frac{1}{b} = \frac{1}{c}$$
$$\frac{2}{a} = \frac{1}{c} - \frac{1}{b}$$
$$\frac{2}{a} = \frac{b}{bc} - \frac{c}{bc} \quad \text{◂ Convert into a common denominator}$$
$$\frac{2}{a} = \frac{b-c}{bc}$$
$$2bc = a(b-c)$$
$$\therefore \quad \underline{\underline{a = \frac{2bc}{b-c}}}$$

Classwork 9.14 Solution

Make v the subject of the formula $\frac{2}{u} - \frac{1}{v} = \frac{1}{f}$.

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Skills Upgrading Corner 9.4

Solution

Make the letter in the brackets the subject of each of the following formulae.

(a) $B = 90(s+2)$ [s]

(b) $\dfrac{5d+7}{2} = e$ [d]

(c) $4E = 6g - mg$ [g]

(d) $(3s+2)x + 5s = 8$ [s]

Exercise 9D

Level 1

Make the letter in the brackets the subject of each of the following formulae. (**1** – **7**)

1. (a) $a+b+c=180$ [a] **(b)** $V = \ell bh$ [ℓ]

2. (a) $S = \dfrac{1}{3}Ah$ [h] **(b)** $b^2 = 4ac$ [a]

3. (a) $y = 4x - 1$ [x] **(b)** $y = 2 - 3x$ [x]

4. (a) $K = (b-2)A + a$ [A] **(b)** $s = \dfrac{1}{2}(u+v)t$ [t]

5. (a) $m = \dfrac{1}{2}(a-b)$ [a] **(b)** $S = 180(n-2)$ [n]

6. (a) $a = \dfrac{6-b}{5}$ [b] **(b)** $A = \dfrac{a}{1-r}$ [r]

7. (a) $\dfrac{m}{2} = \dfrac{2n+3}{3}$ [n] **(b)** $y - 1 = \dfrac{4x-1}{2}$ [x]

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Level 2

Make the letter in the brackets the subject of each of the following formulae. (**8** – **13**)

8. **(a)** $R = sa - sk$ [s] **(b)** $Da + Eb = Ab$ [b]

9. **(a)** $\dfrac{b}{a} - \dfrac{1}{a} = 4$ [b] **(b)** $\dfrac{f}{h} - dm = m$ [m]

10. **(a)** $G = \dfrac{m+1}{m}$ [m] **(b)** $B = \dfrac{2t}{t-2}$ [t]

11. **(a)** $V = \dfrac{1}{6}h(3r + h^2)$ [r] **(b)** $x = \dfrac{a+kb}{1+k}$ [k]

12. **(a)** $y + 2 = \dfrac{3x+1}{x}$ [x] **(b)** $(p+r)a + (p-r)b = 0$ [p]

13. **(a)** $p = \dfrac{1}{5q} - \dfrac{1}{r}$ [r] **(b)** $\dfrac{1}{abc} + \dfrac{1}{b} = \dfrac{1}{c}$ [a]

14. It is given that $x = \dfrac{1}{n+1}$ and $y = \dfrac{n}{n-1}$.

(a) **(i)** Express n in terms of x.

(ii) Express n in terms of y.

(b) **(i)** Express y in terms of x.

(ii) Express x in terms of y.

15. In the process of designing the structure of a stadium, the relation among the time for *clearing the crowd* (t minutes), the number of people in the stadium (N), the number of passages (n) and the width of each passage (w m) are considered, and the following formula is obtained:

$$t = 3 + \frac{N}{8nw}$$

(a) It is known that at the end of a match, there are 7 200 people leaving the stadium. If 10 passages of 6 m wide each are opened up for people to pass through, how long will it take to clear all the people?

(b) Make n the subject of the formula.

(c) Police will open up 21 passages of 6 m wide each for clearing the crowd after the count-down on a New Year's Eve. Can 12 000 people be cleared within 15 minutes? Explain your answer briefly.

clearing the crowd 疏散人群

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Chapter Summary

Chapter Summary

Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

Column A	Column B
1. Algebraic fraction •	• **(a)** A variable in a formula which is expressed in terms of the other variables.
2. Formula •	• **(b)** An algebraic equality to express the relation among variables.
3. Subject of a formula •	• **(c)** $\dfrac{P}{Q}$, where P and Q are polynomials and Q involves variables.

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. Simplify the following expressions. 9.1B

(a) $\dfrac{6x^2}{xy}$ **(b)** $\dfrac{9r^2-15r}{3r-5}$

2. Simplify the following expressions. 9.1C

(a) $\dfrac{4x^2}{y^2}\times\dfrac{y^3}{2x}$ **(b)** $\dfrac{c+5d}{7e}\div\dfrac{2c+10d}{14e}$

3. Simplify the following expressions. 9.2

(a) $\dfrac{2}{3x}-\dfrac{1}{6x}$ **(b)** $\dfrac{y}{y-1}-\dfrac{1}{1-y}$

4. It is given that $A=P(1+RT)$. 9.3B

(a) If $P=120$, $R=0.125$ and $T=2$, then $A=$ ____________ .

(b) If $A=320$, $R=0.2$ and $T=3$, find the value of P.

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5. (a) Make x the subject of the formula $y = 25 - 4x$, then 9.4

$x =$ ______________.

(b) Make h the subject of the formula $S = \pi r^2 + 2\pi rh$.

Revision Exercise 9

Level 1

Simplify the following expressions. (**1** – **2**)

1. (a) $\dfrac{36x^2y}{8x}$ (b) $\dfrac{12x^2 + 4y^2}{4xy}$ (c) $\dfrac{ax + axy}{3b + 3by}$

2. (a) $-\dfrac{3}{x-4} + \dfrac{1}{x-4}$ (b) $-\dfrac{x}{x+y} - \dfrac{x+2y}{x+y}$ (c) $\dfrac{y}{-y+x} - \dfrac{x}{x-y}$

3. Given that $x = \dfrac{1}{5}$, $y = 15$ and $z = -3$, find the value of w in each of the following formulae.

(a) $w = -\dfrac{5xyz}{9}$ (b) $w = \dfrac{y^2 - z^2}{xyz}$ (c) $w = \dfrac{-5z + y}{10} - 25x^2$

4. Make the letter in the brackets the subject of each of the following formulae.

(a) $y = \dfrac{x}{3} + 4$ [x] (b) $2p + 3q - 4r = 0$ [q] (c) $C = P(1 + \dfrac{RT}{100})$ [T]

Level 2

Simplify the following expressions. (**5** – **6**)

5. (a) $\dfrac{3x-15}{7x+21} \times \dfrac{2x+6}{5x-x^2}$ (b) $\dfrac{1-2p}{5q+15p} \div \dfrac{14p-7}{q+3p}$ (c) $\dfrac{-6x^2+16xy}{3y-6x} \div \dfrac{32y-12x}{5y^2-10xy}$

6. (a) $\dfrac{2}{3a} + \dfrac{3}{a}$ (b) $\dfrac{7}{12(b-1)} - \dfrac{3}{4(b-1)}$ (c) $\dfrac{2c}{2(c-d)} - \dfrac{d}{8(c-d)}$

7. Given that $y = \dfrac{4}{3(2-x)}$, find the value of x if $y = 2$.

8. It is given that $\dfrac{3}{c} = \dfrac{2}{a} - \dfrac{6}{b}$. If $c = 12$ and $a = 9$, find the value of b.

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9. Given that $\frac{9}{q}=\frac{8p}{3p+2}$, find the value of p if $q=30$.

10. Make the letter in the brackets the subject of each of the following formulae.

(a) $\frac{1}{a}-\frac{1}{b}=\frac{3}{b}$ $\quad$ [b]

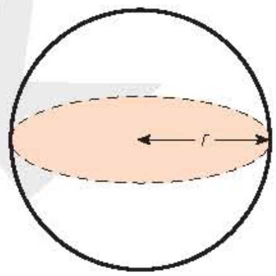
(b) $y=\frac{z}{10x}-\frac{1}{2x}$ $\quad$ [x]

(c) $q=\frac{2}{rp}-\frac{1}{p}$ $\quad$ [p]

11. The surface area A of a sphere is given by the following formula:

$$A=4\pi r^2$$

where r is the radius of the sphere.



(a) Given the following, find the value of A.

(i) $\pi=3.14$ and $r=1$

(ii) $\pi=\frac{22}{7}$ and $r=3.5$

(b) Make r^2 the subject of the formula.

(c) For each of the following values of A, find the value of r.

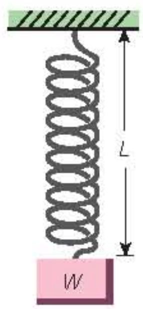
(i) 36π

(ii) 196π

12. Without hanging anything, the length of a *spring* is ℓ. Its length L after hanging something is given by the formula:

$$L=\ell+\frac{W}{k}$$

where W is the weight of the hanging object and k is a constant.



(a) **(i)** Make W the subject of the formula.

(ii) Find the value of W when $L=13$, $\ell=8$ and $k=1.5$.

(b) **(i)** Make k the subject of the formula.

(ii) Find the value of k when $L=30$, $\ell=26$ and $W=4$.

13. Simplify the following expressions.

(a) $\frac{x}{2x+y}\div\frac{x+3y}{10x+5y}\times\frac{x^2-9y^2}{4x}$

(b) $(2m+5)\div\frac{(5+2m)^2}{(3-n)^2}\times\frac{2m+5}{3m-mn}$

(c) $\frac{4mn}{3m-2}-5n$

(d) $\frac{-3a-9}{(a+3)(a-3)}+\frac{4}{a-3}$

spring 彈簧

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14. Make x the subject of each of the following formulae.

(a) $y=\dfrac{2x+3}{x-1}$

(b) $\dfrac{xy-1}{y}=1-\dfrac{2x}{y}$

15. It is given that the sum of the squares of all integers from 1 to n is $S=\dfrac{n(n+1)(2n+1)}{6}$.

(a) Find $1^2+2^2+\cdots+10^2$.

(b) Find $1^2+2^2+\cdots+20^2$.

(c) Find $11^2+12^2+\cdots+20^2$.

16. It is given that the sum of the cubes of all integers from 1 to n is $S=[\dfrac{n(n+1)}{2}]^2$. Find $7^3+8^3+\cdots+16^3$.

17. (a) Simplify $\dfrac{y}{3x}+\dfrac{y}{2x}-\dfrac{y}{x}$.

(b) From the result of (a), make y the subject of the formula $\dfrac{y}{3x}+\dfrac{y}{2x}-\dfrac{y}{x}=\dfrac{y+1}{2x-1}$.

(c) Consider the formula $\dfrac{y}{3x}+\dfrac{y}{2x}-\dfrac{y}{x}=\dfrac{y+1}{2x-1}$. If $x=\dfrac{1}{5}$, find the value of y.

MC Question

18. Simplify $\dfrac{2cd+8de}{4df}$.

A. $\dfrac{4c+e}{2f}$

B. $\dfrac{2c+e}{4f}$

C. $\dfrac{c+2e}{4f}$

D. $\dfrac{c+4e}{2f}$

☐

19. If $\dfrac{2a^2}{6ac}\times\dfrac{9ac^2}{k}=3a^2$, then $k=$

A. $\dfrac{1}{c}$.

B. c.

C. ac.

D. $3a^2$.

☐

20. $\dfrac{9x^2}{4y}\div\dfrac{3xy}{8y^2}=$

A. $\dfrac{27x^3}{32y^2}$.

B. $\dfrac{6x}{y}$.

C. $6x$.

D. x.

☐

21. Simplify $\dfrac{u}{u-v}+\dfrac{2v}{v-u}$.

A. $\dfrac{u+2v}{u-v}$

B. $\dfrac{u+2v}{v-u}$

C. $\dfrac{u-2v}{u-v}$

D. $\dfrac{u-2v}{v-u}$

☐

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22. It is given that $2w = \frac{3t+e}{5}$. If $w = 5$ and $e = 8$, find the value of t.

A. 14

B. 12

C. $\frac{10}{3}$

D. $\frac{2}{3}$

23. It is given that $\frac{1}{x} = \frac{4y-7}{8}$. Which of the following is correct?

A. $y = \frac{8x+7}{4}$

B. $y = \frac{7x+8}{4}$

C. $y = \frac{7x+8}{4x}$

D. $y = \frac{7x+2}{x}$

Problem-solving and Exploring

Hint for the Title Page Question

(a) Simplify the following questions and find the patterns.

When $n = 2$, $T =$ ______________________________;

When $n = 3$, $T =$ ______________________________;

When $n = 4$, $T =$ ______________________________;

When $n = 5$, $T = 10$.

(b) Find that $T = 10$ when $n = 5$ according to the following.

$$\begin{array}{rr} & 1 + 2 + 3 + 4 \\ +) & 4 + 3 + 2 + 1 \\ \hline \end{array}$$

(c) Figure out the relation between T and n according to the following.

$$\begin{array}{rccccccccccccc} & 1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & (n-2) & + & (n-1) \\ +) & (n-1) & + & (n-2) & + & (n-3) & + & (n-4) & + & \cdots & + & 2 & + & 1 \\ \hline \end{array}$$

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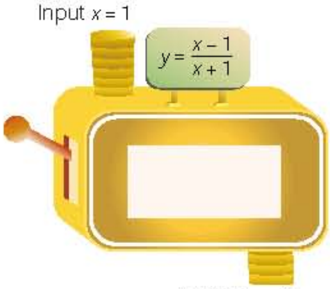
Additional Question

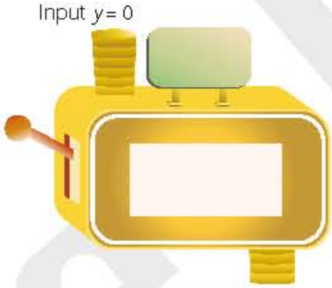
1. (a) A formula is given by $y = \dfrac{x-1}{x+1}$, find the value of y for the following values of x.
   (i) 0
   (ii) 3
   (iii) 5

   (b) The formula $y = \dfrac{x-1}{x+1}$ is saved in number machine A. When any number x is input, a number y will come out as an output.



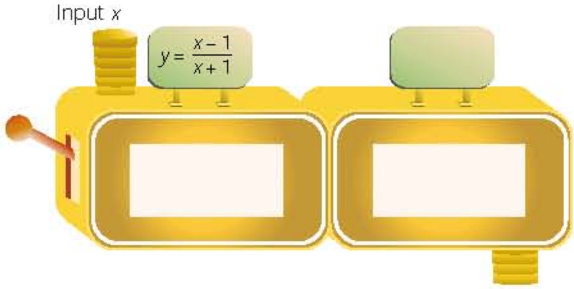
Number machine A

   (i) Another formula is saved in number machine B, under which all output y from number machine A can be restored to the original number x. Find the formula.



Number machine B

   (ii) Number machine C is a combination of number machines A and B. What is the output from number machine C when 0, 3 and 5 are input?



Number machine C

   (iii) What output will you get when a number is input to number machine C?

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2. The *dosage* of medicine for a child is usually less than that for an adult. A manufacturer calculates the dosage of a medicine for a child using the formula $C=\frac{y}{y+12}\times a$, where C stands for the dosage of a child, y stands for the age of a child under 13 and a stands for the dosage for an adult.

(a) For children of the following ages, what fraction of an adult's dosage does each of them need?

(i) 1 year old

(ii) 5 years old

(iii) 9 years old

(b) If a child needs half of an adult's dosage, how old is the child?

dosage 劑量

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Exploration with Fun — Enrichment

Linear Literal Equations in One Unknown

We often come across linear equations in one unknown, e.g.

$$8x-5=0$$

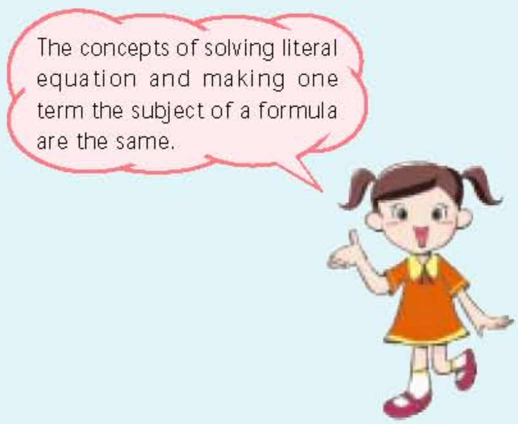
If the numerals of the equation are replaced by letters, the equation can turn into a literal equation. Thus, the above equation can be written as

$$ax-b=0$$

The methods of solving literal and simple equations are similar.

	Simple equation	Literal equation
(a)	Find x in the equation $8x-5=0$ $8x=5$ $\therefore\quad x=\underline{\underline{\frac{5}{8}}}$	Find x in the equation $ax-b=0$ $ax=b$ $\therefore\quad x=\underline{\underline{\frac{b}{a}}}$
(b)	Find x in the equation $3x+4=6-5x$ $3x+5x=6-4$ $8x=2$ $\therefore\quad x=\underline{\underline{\frac{1}{4}}}$	Find x in the equation $ax+b=d-cx$ $ax+cx=d-b$ $(a+c)x=d-b$ $\therefore\quad x=\underline{\underline{\frac{d-b}{a+c}}}$
(c)	Find x in the equation $\frac{x}{5}+\frac{x}{7}=3$ $35(\frac{x}{5}+\frac{x}{7})=3\times35$ $7x+5x=105$ $12x=105$ $\therefore\quad x=\underline{\underline{\frac{35}{4}}}$	Find x in the equation $\frac{x}{a}+\frac{x}{b}=c$ $ab(\frac{x}{a}+\frac{x}{b})=abc$ $bx+ax=abc$ $(b+a)x=abc$ $\therefore\quad x=\underline{\underline{\frac{abc}{b+a}}}$

◂ Group the like terms.



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Extension 9.1

Solution

1. Solve the following equations in x.

   (a) $x + 2 = 3a$

   (b) $b + 3x = 2c$

   (c) $x + b = ax - c$

   (d) $p - 3x = 2(x + q)$

   (e) $\frac{x}{c} + \frac{x}{d} = e$

   (f) $\frac{a}{x} - \frac{b}{x} = 3a + b$

Solution

2. There are x apples, which can be evenly distributed to group A of m children or group B of n children. It is known that each child of group A can obtain p more apples than group B.

   (a) Set up an equation in x by using m, n and p.

   (b) Compare m and n.

   (c) It is given that $m = 2$, $p = 11$, n and x are positive integers, and n is less than 8. Find one set of possible values of n and x.

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